Testing for Fractional integration versus short memory with trends and structural breaks.*

Laura Mayoral
Dept. of Economics and Business
Universidad Pompeu Fabra.

Abstract

Although it is commonly accepted that most macroeconomic variables are non-stationary, it is often difficult to identify the source of the non-stationarity. In particular, integrated processes and short memory models containing trending components share some statistical properties and this makes their identification a hard task. The problem gets even harder in the presence of parameter instability. The goal of this paper is to extend the classical testing framework of I(1) versus I(0)+trends and/or breaks by considering a more general class of models under the null hypothesis: non-stationary fractionally integrated (FI) processes. The proposed test is developed in the time domain and is very simple to compute. The asymptotic properties of the new technique are derived and it is shown by simulation that it is very well-behaved in finite samples. To illustrate the usefulness and the simplicity of the proposed technique, an application using inflation data is also provided.

Keywords: fractional integration, structural breaks, unit roots, trends.
JEL Classification: C22, C12

1. Introduction

A standard practice in most macroeconomic applications is to test whether the trend component of a variable is best represented as stochastic or deterministic. Typically, the former is captured by processes containing a unit root while the latter is represented as the sum of a stochastic short-memory component and some deterministic trends. Perron (1989) contributed to this literature by showing that standard unit root tests could lead to erroneous conclusions if the true DGP was a short memory $-I(0)-$ process containing breaks in the deterministic components. This seminal contribution was the starting point of a myriad of articles on the problem of distinguishing between $I(1)$ vs. $I(0)+breaks$.

*I would like to thank to Jushan Bai, Javier Hidalgo, Peter Robinson and other participants in seminars at LSE, CORE and UB. I acknowledge financial support from the Spanish Ministry of Education through grants SEC2003-04429 and SEC2003-04476 and also from the Barcelona Economics Program of CREA.

Nevertheless, unit root processes are a very particular class within the group of integrated processes that can be used to represent stochastic trends. Fractionally integrated (FI) processes contain the former as a particular case but, by allowing for a fractional order of integration, are able to represent a richer class of behaviors (see Baillie, (1996)). Furthermore, there is abundant evidence that supports the empirical relevance of these class of models in macroeconomics and finance, justified from both a theoretical and an applied perspective (cf. Henry and Zaffaroni (2002) for a list of references on this subject).

Not surprisingly, it is well-known that it is also difficult to provide an unambiguous answer as to whether a process is best represented as fractionally integrated or as short memory plus some deterministic components, possibly perturbed by sudden changes, since a similar identification problem as in the I(1) case holds here. The issue of detecting patterns similar to those of a FI process when the DGP is short memory containing deterministic terms and/or breaks has been widely analyzed (cf. Battacharya et al. (1983), Künsch (1986), Teverovsky and Taqqu (1997), Giraitis et al. (2001), Diebold and Inoue (2001), Perron and Qu (2004) and Davidson and Sibbertsen (2005), among others).[1] It is generally concluded that the use of standard techniques devised for FI processes could lead to the detection of spurious persistence when applied on short memory processes containing trends and/or breaks. The opposite effect is also well-documented, that is, conventional procedures for detecting and dating structural changes tend to find spurious breaks, usually in the middle of the sample, when in fact there is only fractional integration in the data (see Nunes et al. (1995), Krämer and Sibbertsen (2002) and Hsu (2001)).

There is an increasing interest on developing techniques that are able to distinguish between fractional integration and I(0) models containing trends and/or breaks. Most of them consider the problem of testing for (stationary) long memory versus a weakly dependent series with monotonic trended components or breaks in the mean (see Künsch (1986), Heyde and Dai (1996), Sibbertsen and Venetis (2004) and Berkes et al. (2005) , among others). Although this problem is of genuine interest, it is often not useful in many macroeconomic applications where variables are markedly non-stationary. There are fewer contributions that are able to deal with nonstationary values of d, Dolado et al. (2005) and Shimotsu (2005) being notable exceptions.

The goal of this paper is to develop a simple testing device that is able to determine whether the non-stationarity observed in the data is due to strong persistence of the shocks, modelled as a non-stationary fractionally integrated process, or to the existence of deterministic trends, possibly containing breaks,

[1] The latter authors also point out that cross-sectional aggregation of a fairly general class of nonlinear processes produces a model that not only has the same correlation patterns as FI processes but is also observationally equivalent to FI, in the sense that the aggregated model is linear and converges to fBM.

in an otherwise short memory process. The test is developed in the time domain and is very simple to compute. Besides, it has a semiparametric character and then it is not needed to model the short-term autocorrelation of the process. The asymptotic behavior of the test is provided and its finite sample performance is analyzed via Monte Carlo simulations. Appropiate comparisons with other competing techniques are also provided.

The structure of the paper is as follows. Section 2 presents the model and the hypotheses of interest. Section 3 analyzes the problem of testing for FI vs. $I(0)+$ trended regressors. This framework is extended in Section 4 by allowing for the presence of breaks (occurring at an unknown time) in the deterministic components. The output of some Monte Carlo simulations that evaluate the performance of the test in finite sample is reported in Section 5. To illustrate the simplicity and empirical usefulness of this technique, an application using inflation data has been included. Section 7 draws some final conclusions. All proofs are gathered in Appendix A while critical values for the proposed tests are presented in Appendix B.

In the sequel, the definition of a $FI\,(d)$ process that we will adopt is that of an (asymptotically) stationary process, when $d < 0.5$ and that of a non-stationary (truncated) process, when $d \geq 0.5$. Those definitions are similar to those used in, e.g., Robinson (1994) or Tanaka (1999) (see, Appendix A in Dolado, Gonzalo and Mayoral (2002) for details). Moreover, the following conventional notation is adopted throughout the paper: L is the lag operator, $\Delta = (1-L)$, $\Gamma(.)$ denotes the gamma function, $\{\pi_i(d)\}$ represents the sequence of coefficients associated to the expansion of Δ^d in powers of L and are defined as

$$\pi_i(d) = \frac{\Gamma(i-d)}{\Gamma(-d)\,\Gamma(i+1)}. \tag{1}$$

All integrals are taken with respect to the Lebesgue measure; $B_d(.)$ is standard fractional Brownian motion (fBM) corresponding to the limit distribution of the standardized partial sums asymptotically stationary (truncated) $FI\,(d)$ processes;[2] Finally, $\xrightarrow{w}$ and $\xrightarrow{p}$ denote weak convergence and convergence in probability, respectively.

2. The model and the hypotheses

In the following it is assumed that the data $y_1, ..., y_T$ is generated as,

$$y_t = \beta' Z_t + \delta' V_t(\omega) + x_t, \; t = 1, 2, ..., \tag{2}$$

where,

$$\begin{aligned} \Delta^d x_t &= u_t, t = 1, 2, ... \qquad (3)\\ x_t &= 0 \text{ for all } t \leq 0, \qquad (4) \end{aligned}$$

[2] According to the notation introduced in Marinucci and Robinson (1999) $B_d(.)$ is a type II Fractional Brownian motion

and,

$$V_t(\omega) = \begin{cases} Z_{t-T_B} & t \geq T_B, \\ 0 & otherwise. \end{cases}$$

The process y_t and the $k \times 1$ vector Z_t of non-stochastic variables are observable,[3] β and δ are $k \times 1$ vectors or parameters and $\{u_t\}$ is an unobserved zero-mean process whose spectral density is strictly positive at zero frequency. Moreover, $\{u_t\}$ is assumed to have a Wold representation,

$$u_t = \Psi(L)\,\varepsilon_t, \tag{5}$$

where the coefficients ψ_j are such that $\sum_{j=0}^{\infty} j\left|\psi_j\right| < \infty$ and $\{\varepsilon_t\}$ is an unobserved *i.i.d.* zero mean process with unknown variance equal to σ^2 and $\mu_4 = E|\varepsilon|^4 = \eta\sigma^4 < \infty$. T_B is the (unknown) parameter describing the time when the break, if it exists, occurs and $\omega = T_B/T$ determines the location of the break point in the sample. It verifies that

$$\omega \in \Omega = [\omega_L, \omega_H] \subset (0,1).$$

Since the objective of this paper is to determine the source of the non-stationarity observed in the data, more specifically, whether it comes from a high degree of inertia or the existence of (possibly unstable) trended components in an otherwise short-memory process, the non-stationarity is modelled in two different ways. Under the null hypothesis y_t is considered to be a non-stationary $FI(d)$ process with no breaks so that δ is assumed to be equal to zero (no breaks). In this case, it will be assumed that y_t is $FI(d)$ with $d \in (1/2, 3/2)$.[4] This assumption can be easily relaxed by using the results in Marinucci and Robinson (1999) but we concentrate on this range since it is the most relevant in applications with nonstationary series. In some economic problems, the value of d under the null hypothesis is known, for instance, in the popular unit root case where d is set equal to 1. Nevertheless, in most relevant situations it is unknown. Accordingly, the null hypothesis will be simple or composite, respectively, that is,

$$H_0 \ : \ d = d_0,\ \delta = 0 \text{ for some } d_0 > 1/2, \tag{6}$$

$$\text{or} \quad H_0^{'} \ : \ d \in D_0,\ \delta = 0,\ D_0 \subset (1/2, 3/2) \tag{7}$$

corresponding to the cases where the order of integration under H_0, d_0, is known or unknown, respectively. Under H_1, y_t is short memory and therefore $d = 0$ is imposed. The case where the alternative hypothesis is a short memory process plus some trended components (without breaks) is analyzed in

[3] Z_t will typically contain polynomials of t.

[4] The range of nonstationary values of d also includes the value $d = 1/2$. However, this value is excluded in the present analysis since it constitutes a discontinuity point in the asymptotic theory.

Section 3. Thus, in this section it is assumed that $\delta = 0$ also under H_1. Section 4, in turn, deals with the case where δ is (partially or totally) unrestricted, allowing in this way for the possibility of breaks occurring at an unknown time T_B. More specifically the alternative hypothesis is formulated as,

$$H_1 \ : \ d = 0, \ \delta = 0, \tag{8}$$

$$\text{or} \qquad H_1^{'} \ : \ d = 0, \ \delta \text{ unrestricted (totally or partially)}, \tag{9}$$

that will be analyzed in Sections 3 and 4, respectively. In the latter case, attention is restricted, for simplicity, to the case where there exists at most a single break. An extension to a multiple-change environment can be entertained along the lines of Bai (1999) and Bai and Perron (1998).

It will not be needed to make additional parametric assumptions on the structure of u_t but, if u_t admits an ARMA(p,q) representation, y_t will be an ARFIMA (p, d, q) process under H_0, (see Hosking, (1981) and Granger and Joyeux, (1980)) or a trend-stationary ARMA(p,q) model under H_1.

3. Testing for non-stationary fractional integration versus short memory and trends

The literature on testing whether a process posses a unit root or is best represented as weakly dependent plus some deterministic trends is immense (see Xiao and Phillips, (1998) for a survey). There is also an increasing interest on the problem of distinguishing between (stationary) long memory and I(0) processes containing trended components (cf. Künsch (1986), Heyde and Dai (1996), Sibbertsen and Venetis (2004) and Berkes et al. (2005)). Nevertheless, the problem of testing for non-stationary fractional integration versus short memory + trends has been largely overlooked in the literature. This is very surprising given the enormous number of contributions devoted to the study of the I(1)-I(0) + trends identification problem and that the I(1) model is only a very particular class within the group of non-stationary, highly persistent processes. Section 5 includes some simulations that illustrate the identification problem addressed in this section: standard methods for estimating FI processes find values of d in the non-stationary interval when the true DGP is $i.i.d$ + some trends.

In this section we deal with the problem of testing H_0 and H_0' defined in (6) and (7), against the alternative hypothesis of short memory+trends, as described in (8). Consider first the case where the value of d under H_0, d_0, is known. This problem is quite simple since it just becomes a test of simple hypotheses. Then, it is possible to derive the most powerful invariant test. Assuming gaussianity, minus two times the log-likelihood function under H_0 and H_1 is given, respectively, by (except for an additive constant),

$$L\left(d, \Sigma, \beta\right)|_{H_0} = \left(\Delta^{d_0} y - \Delta^{d_0} Z \beta_0\right)' \Sigma^{-1} \left(\Delta^{d_0} y - \Delta^{d_0} Z \beta_0\right),$$

and

$$L\left(d, \Sigma, \beta\right)|_{H_1} = \left(y - Z\beta_1\right)' \Sigma^{-1} \left(y - Z\beta_1\right),$$

respectively, where $\Delta^{d_0} y = \left(\Delta^{d_0} y_2, ..., \ \Delta^{d_0} y_T\right)'$, $y = (y_2, ..., \ y_T)'$, $Z = (Z_2', ..., \ Z_T')'$, $\Delta^{d_0} Z = \left(\Delta^{d_0} Z_2', ..., \ \Delta^{d_0} Z_T'\right)$ and Σ is the non-singular variance-covariance matrix of $(u_2, ..., u_T)$.

From the developments in Lehmann (1959), the most powerful invariant test of $d = d_0$ vs. $d = 0$ rejects H_0 for small values of

$$\min_{\beta} (y - Z\beta)' \Sigma^{-1} (y - Z\beta) - \min_{\beta} \left(\Delta^{d_0} y - \Delta^{d_0} Z\beta\right)' \Sigma^{-1} \left(\Delta^{d_0} y - \Delta^{d_0} Z\beta\right). \quad (10)$$

The test statistic is the difference in weighted sum of squared residuals from two constrained GLS regressions, one imposing $d = d_0$ and the other $d = 0$. This test is not feasible since Σ is in general unknown. However, it is possible to obtain a feasible test with the same large sample properties. A first approach would be to replace Σ by an appropriate estimate. But to do so it would be necessary to impose some parametric restrictions on the correlation structure of u_t. We advocate for a semiparametric approach and leave the parametric structure of u_t unspecified. If $u_t = \varepsilon_t$ is an i.i.d sequence, then $\Sigma = \sigma^2 I_{T-1}$, where I_{T-1} is the identity matrix of dimension $(T-1)$. Substituting σ^2 by a consistent estimate under H_0 and rearranging terms, it follows that the test rejects H_0 for small values of,[5]

$$R(d_0) = \frac{\left(y - Z\hat{\beta}_1\right)' \left(y - Z\hat{\beta}_1\right)}{\left(\Delta^{d_0} y - \Delta^{d_0} Z\hat{\beta}_0\right)' \left(\Delta^{d_0} y - \Delta^{d_0} Z\hat{\beta}_0\right)}. \quad (11)$$

where $\hat{\beta}_0$ and $\hat{\beta}_1$ are the OLS estimators under H_0 and H_1, respectively. The statistic in (11) can also be used even if u_t displays autocorrelation and the same distribution as in the uncorrelated case can be obtained as long as a non-parametric correction that accounts for the autocorrelation in u_t is introduced. Theorem 1 below describes the asymptotic distribution of the test statistic when u_t is a general linear weakly dependent process and Z is a polynomial function of t. The asymptotic behavior depends on the components included in the matrix Z. Explicit formulas for the popular case where Z contains a constant or a constant a linear time trend are provided (see Appendix A) but it can be easily generalized to higher order polynomials and other exponential functions of t. It turns out that, if an appropiate correction that accounts for the correlation in u_t is introduced, the test is asymptotically equivalent to the most powerful invariant test described in (10).

Theorem 1 *Let y_t be a $FI(d_0)$ process as defined in* (2) *for some $d_0 \in (1/2, \ 3/2)$ with Z_t=(1) or*

[5] The statistic in (11) is similar to the Von-Neumann ratio proposed in the framework of efficient unit root tests (see Sargan and Bhargava (1983) and Bhargava (1986)). These authours showed that the statistic was locally most powerful for testing the hypotheses of a random walk versus an AR(1) process. Schmidt and Phillips (1992) showed that the Lagrange multiplier principle also leads to a similar expression for a Gaussian likelihood.

$Z_t = \begin{pmatrix} 1 & t \end{pmatrix}$ and $\delta = 0$. Then,

$$T^{1-2d_0} \frac{\left(y - Z\hat{\beta}_1\right)' \left(y - Z\hat{\beta}_1\right)}{\left(\Delta^{d_0} y - \Delta^{d_0} Z\hat{\beta}_0\right)' \left(\Delta^{d_0} y - \Delta^{d_0} Z\hat{\beta}_0\right)} \overset{w}{\rightarrow} \left(\frac{\lambda^2}{\gamma_0}\right) \int_0^1 \left(B_{d_0}^Z (r)\right)^2 dr.$$

where $\lambda = \sigma\Psi(1)$, $\gamma_0 = \sigma^2 \sum_{i=0}^{\infty} \psi_j^2$ and $B_{d_0}^Z(r)$ is the L_2 projection residual from the continuous time regression,

$$B_{d_0}(r) = \hat{\beta}' z(r) + B_{d_0}^z(r, \omega),$$

where $z(r) = 1$ or $(1, r)$ according to whether Z contains a constant or a constant and a linear trend. (See Appendix A for details).

In applications, one should first compute the statistics $R^{\mu}(d_0)$ or $R^{\tau}(d_0)$, according to whether Z is considered to contain a constant or a constant and a trend, defined as

$$R^{\mu}(d_0) = T^{1-2d_0} \left(\frac{\hat{\lambda}^2}{\hat{\gamma}_0}\right)^{-1} \frac{\sum_{t=1}^{T}(y_t - \hat{\alpha}_1)^2}{\sum_{t=2}^{T}(\Delta^{d_0}(y_t - \hat{\alpha}_0))^2}, \tag{12}$$

and

$$R^{\tau}(d_0) = T^{1-2d_0} \left(\frac{\hat{\lambda}^2}{\hat{\gamma}_0}\right)^{-1} \frac{\sum_{t=1}^{T}\left(y_t - \hat{\alpha}_1 - \hat{\beta}_1 t\right)^2}{\sum_{t=2}^{T}\left(\Delta^{d_0}(y_t - \hat{\alpha}_0 - \hat{\beta}_0 t)\right)^2} \tag{13}$$

where $\hat{\alpha}_i$ and $\hat{\beta}_i$, $i = \{0, 1\}$ are the OLS estimators under the corresponding hypotheses. Notice that under H_0, the filter $\Delta^{d_0} 1_{(t>0)}$ should be applied prior to the estimation of $\hat{\alpha}_0$ and $\hat{\beta}_0$. Then, these parameters are computed as the OLS estimates in a regression of $\Delta^{d_0} y_t$ on $\Delta^{d_0} 1_{(t>0)}$ and $\Delta^{d_0 - 1} 1_{(t>0)}$ (or equivalently, $\Delta^{d_0} t$), where $\Delta^{\eta} 1_{(t>0)} = \sum_{i=0}^{t-1} \pi_i(\eta)$ and the coefficients $\pi_i(.)$ are defined in (1). Under H_1, in turn, $\hat{\alpha}_1$ and $\hat{\beta}_1$ are the OLS estimates of y_t on a constant or a constant and a trend for R^{μ} and R^{τ}, respectively.

As a second step, if autocorrelation in u_t is suspected, the term $\left(\hat{\lambda}^2 / \hat{\gamma}_0\right)^{-1}$ should be computed. This factor can be estimated by nonparametric kernel techniques, analogous to those used in the estimation of the spectral density (see Andrews (1991)). More specifically the variance of u_t, γ_0, can be estimated under H_0 by $\sum \left(\Delta^{d_0} y_t - \Delta^{d_0} Z\hat{\beta}_0\right)^2 / T$ whereas λ^2 can be rewritten as:

$$\lambda^2 = \gamma_0 + 2 \sum_{i=1}^{\infty} \gamma_i = 2\pi s_u(0).$$

Several estimators of this quantity have been proposed, see Andrews (1991) for an analysis and comparison of the different techniques. One of the most popular is the Newey-West estimator:

$$\hat{\lambda}^2 = \hat{\gamma}_0 + 2\sum_{i=1}^{q} (1 - j/(q+1)) \hat{\gamma}_i$$

where $\hat{\gamma}_i = T^{-1} \sum_{t=j+1}^{T} u_t u_{t-j}$. Andrews (1991) also provides a guideline for choosing the value of the lag truncation, q.[6] See also Cai and Shintani (2005) for further analysis on the estimation of this quantity. As a final step, the null hypothesis will be rejected whenever the values of $R^{\mu}(d_0)$ or $R^{\tau}(d_0)$ are smaller than the critical values reported in Tables $B1$ and $B2$, respectively.

In most applications the order of integration d_0 is unknown and therefore the tests R^{μ} and R^{τ} are not feasible. In these cases, attention will be focused on composite null hypothesis as H_0', that is, whether the memory parameter d belongs to an specific set $D_0 = [\underline{d}, \overline{d}] \subset (0.5, 1.5)$. Then, the above-described test can be employed in this case by replacing the unknown value d_0 in (12) or (13) by an estimate of d under H_0'. The statistics $R^i\left(\hat{d}\right)$, $i = \{\mu, \tau\}$, will be computed as (12) or (13) but replacing d_0 by $\hat{d}$ and H_0' will be rejected for small values of $R^i\left(\hat{d}\right)$. The following theorem states that the asymptotic distribution of the resulting resulting test statistic is the same as that described in Theorem 1 as long as $\hat{d}$ is consistent under H_0'.

Theorem 2 *Let y_t be a $FI(d_0)$ process as defined in (2) with $\delta = 0$ and $d_0 \in D_0 = [\underline{d}, \overline{d}] \subset (1/2, 3/2)$. Let $R^i\left(\hat{d}\right)$ be the statistics defined in (12) or (13) for $i = \{\mu, \tau\}$, respectively, where d_0 has been replaced by $\hat{d} \in D_0$, a consistent estimate of d_0. Then,*

$$R^i(d_0) - R^i\left(\hat{d}\right) = o_p(1).$$

Many estimation methods of d suit well the framework considered in this paper. Since the process is nonstationary under H_0', one can take first differences and then apply any standard parametric or semiparametric method developed for stationary FI processes. Several techniques also allow for directly estimating the series since they provide consistent estimates of d even when the process is nonstationary, see Velasco (1999a, b), Shimotsu and Phillips, (2000) or Shimotsu (2006), among others.

Finally, the following theorem states the consistency of the technique proposed in this section.

Theorem 3 *Let y_t be defined as in (2), with $d = \delta = 0$. Then, the test based on the statistics $R^i(d_0)$, ($R^i\left(\hat{d}\right)$), rejects the hypothesis of H_0 (H_0') with probability approaching 1.*

[6] As Andrews points out, a correct choice of q is very important since the perfomance of these estimators can greatly depend on this choice.

It is easy to check that the test is also consistent if the true process is $FI(d^*)$ with $d^* < 0.5$. In this case both the numerator and the denominator of R^μ and R^τ are $O_p(T)$ since they contain the sum of squared residuals from two stationary processes. Then, the statistic is the product of T^{1-2d_0}, where d_0 is the value of d used as null hypothesis, and a term that is $O_p(1)$. It follows that the product tends to zero at a rate T^{1-2d_0}, implying that the probability of rejecting H_0 tends to 1.

4. Testing Fractional Integration versus Structural Breaks

In this section the assumption of $\delta = 0$ is relaxed so that breaks in the deterministic components may occur under the alternative hypothesis.

As in Section 3, we consider first the problem of testing H_0 versus H_1'. Let y_t be defined as in (2) where d_0 is known and δ is totally or partially unrestricted under H_1'. If the time when the break takes place was known, the vector $V_t(\omega)$ would be completely determined and then, the most powerful invariant test would reject the null hypothesis of fractional integration for small values of $\inf_{\beta,\delta} L(d = 0, \Sigma, \beta, \delta) - \inf_{\beta} L(d = d_0, \Sigma, \beta, \delta = 0)$. Under Gaussianity, the critical region of this test would be given by,

$$\inf_{\beta,\delta} (y - Z\beta - V\delta)' \Sigma^{-1} (y - Z\beta - V\delta) - \inf_{\beta} \left(\Delta^{d_0} y - \Delta^{d_0} Z\beta\right)' \Sigma^{-1} \left(\Delta^{d_0} y - \Delta^{d_0} Z\beta\right) < k_T, \qquad (14)$$

for some k_T. Although this test is unfeasible for the reasons stated in Section 3, it would be possible to devise a statistic with analogous large sample properties just by following the same lines as in Section 3. However, this is not the case if the date of the break T_B is unknown because the parameter T_B is not identified under H_0. The usual testing approach in this case amounts to first computing the (feasible) test for a large number of different values of T_B, and then computing a certain functional of these pointwise statistics (see for instance Andrews and Ploberger, (1994)).

We follow here this approach by considering the infimum of a sequence of a statistics computed for different values of $T_B/T = \omega \in \Omega \subset (0,1)$. Two distinct situations can arise when defining the subset Ω. The first is when the interest is centered on change points in a known restricted interval, say $\Omega = [\omega_L, \omega_H]$ for $0 < \omega_L < \omega_H < 1$. This would be the case when one wants to test for changes initiated by some institutional or political change that has occurred at a known period. The second is the case where no information is available a priori and hence, all points in $(0,1)$ are of some interest. This situation may arise when one wants to apply a test of structural break as a general diagnostic test of model adequacy. Since considering the whole interval $(0,1)$ would result in tests with very low power, optimization is carried out in $\omega \in \Omega$, where $\Omega = [\omega_L, \omega_H]$ for some $0 < \omega_L < \omega_H < 1$. More specifically, when no information on the location of the break is available, we will use the restricted interval $\Omega = [0.15, 0.85]$, as suggested by Andrews (1993). The test statistic now becomes,

$$R_b(d_0) = T^{1-2d_0} \frac{\inf_{\omega\in\Omega}(\sum_{t=1}^{T}(y_t - \hat{\beta}_1' Z_t - \hat{\delta}_1(\omega)' V_t(\omega))^2)}{\sum_{t=2}^{T}(\Delta^{d_0}(y_t - \hat{\beta}_0' Z_t))^2} \qquad (15)$$

where both $\delta(\omega)$ and $V_t(\omega)$ depend explicitly on ω and $\hat{\beta}_1$, $\hat{\delta}_1(\omega)$ and $\hat{\beta}_0$ are the OLS estimates under H_1 and H_0, respectively. The asymptotic distribution depends upon the regressors contained in Z_t and also on the parameters that are allowed to break. Theorem 4 describes the large sample properties of the statistic in (15). In particular, explicit formulas and critical values are provided for the four cases considered by Perron (1989) and Zivot and Andrews (1992). In three of these models, Z_t contains both a constant and a linear trend but they differ on the parameters that are allowed to break: Model 1 allows for a break in the level of the series, Model 2 allows for a change in the rate of growth and finally, Model 3 admits both changes. In addition, we also consider "Model 0", where Z_t only contains a constant that is allowed to break once in the sample.[7] More specifically,

$$\text{Model 0}: R_b^0(d_0) = T^{1-2d_0} \frac{\inf_{\omega\in\Omega}(\sum(y_t - \hat{\alpha}_1 - \hat{\delta}_1 DC_t)^2)}{\sum\left(\Delta^{d_0}(y_t - \hat{\alpha}_0 - \hat{\beta}_0 t)\right)^2}, \tag{16}$$

$$\text{Model 1: } R_b^1(d_0) = T^{1-2d_0} \frac{\inf_{\omega\in\Omega}(\sum(y_t - \hat{\alpha}_1 - \hat{\delta}_1 DC_t - \hat{\beta}_1 t)^2)}{\sum\left(\Delta^{d_0}(y_t - \hat{\alpha}_0 - \hat{\beta}_0 t)\right)^2}, \tag{17}$$

$$\text{Model 2: } R_b^2(d_0) = T^{1-2d_0} \frac{\inf_{\omega\in\Omega}(\sum(y_t - \hat{\alpha}_1 - \hat{\beta}_1 t - \hat{\delta}_1 DT_t)^2)}{\sum\left(\Delta^{d_0}(y_t - \hat{\alpha}_0 - \hat{\beta}_0 t)\right)^2}, \tag{18}$$

and

$$Model\ 3: R_b^3(d_0) = T^{1-2d_0} \frac{\inf_{\omega\in\Omega}(\sum(y_t - \hat{\alpha}_1 - \hat{\delta}_1 DC_t - \hat{\beta}_1 t - \hat{\delta}_2 DT_t)^2)}{\sum\left(\Delta^{d_0}(y_t - \hat{\alpha}_0 - \hat{\beta}_0 t)\right)^2}, \tag{19}$$

where $\text{DC}_t = 1$, if $t > T_B$ and 0 otherwise and $DT_t = (t - T_B)$ if $t > T_B$ and 0 otherwise.

Theorem 4 *Let y_t be a $FI(d_0)$ process as defined in (2) for some $d_0 \in (1/2, 3/2)$ where Z contains a constant or a constant and a linear trend. The asymptotic distribution of R_b^i for i={0,1,2,3} is given by,*

$$R_b^i(d_0) \xrightarrow{w} \frac{\lambda^2}{\gamma_0} \inf_{\omega\in\Omega}(\int_0^1 (B_{d_0}^i(r,\omega))^2 dr,$$

where $B_{d_0}^i(r,\omega)$ is the L_2 projection residual from the continuous time regressions,

$$\textit{Model 0: } B_{d_0}(r) = \hat{\alpha}_1 + \hat{\delta}_1 dc(\omega, r) + B_{d_0}^0(r,\omega),$$

$$\textit{Model 1: } B_{d_0}(r) = \hat{\alpha}_1 + \hat{\delta}_1 dc(\omega, r) + \beta_1 r + B_{d_0}^1(r,\omega)$$

$$\textit{Model 2: } B_{d_0}(r) = \hat{\alpha}_1 + \hat{\beta}_1 r + \hat{\delta}_1 dt(\omega, r) + B_{d_0}^2(r,\omega),$$

[7] This case may be of interest when modeling series who do not seem to display a trend, such as inflation or interest rates.

$$Model\ 3\text{: } B_{d_0}(r) = \hat{\alpha}_1 + \hat{\delta}_1 dc(\omega, r) + \hat{\beta}_1 r + \hat{\delta}_2 dt(\omega, r) + B^3_{d_0}(r, \omega),$$

where $dc(\omega, r) = 1_{(r>\omega)}$ *and* $dt(\omega, r) = (r - \omega) 1_{(r>\omega)}$.

Critical values of the distributions above have been obtained by Monte Carlo simulation for the uncorrelated case ($u_t = \varepsilon_t$) and are presented in Appendix B. If correlation of u_t is suspected, the nuisance parameters λ^2 and γ_0 can be estimated according to the techniques detailed in Section 3.

When composite hypotheses such as H_0' are considered, an estimate value of d under that hypothesis is needed prior to carry out the tests. Then, the statistics R_b^i defined in (16) to (19) can still be employed for testing H_0' by replacing d_0 by $\hat{d}$. Theorem 5 states that under H_0', if a consistent estimator of d_0 is used to construct $R_b^i\left(\hat{d}\right)$, for $i = \{0,\ 1,\ 2,\ 3\}$, then the same asymptotic distributions as in Theorem 4 are obtained.

Theorem 5 *Let* y_t *be a* $FI(d_0)$ *process as defined in* (2) *and* $d_0 \in D_0 = [\underline{d},\ \overline{d}] \subset (1/2,\ 3/2))$. *Let* $R_b^i\left(\hat{d}\right)$ *be the statistics defined in* (16) *to* (19) *where* d_0 *has been replaced by a consistent estimate,* $\hat{d} \in D_0$. *Then, it holds that*

$$R_b^i(d_0) - R_b^i\left(\hat{d}\right) = o_p(1),\ \text{for } i = \{0,\ 1,\ 2,\ 3\}. \tag{20}$$

It follows that H_0' will be rejected when the value of $R_b^i\left(\hat{d}\right)$ is smaller than the corresponding critical value (provided in Appendix B, Tables B3-B6).

Finally, the following theorem states the consistency of the proposed test.

Theorem 6 *Let* y_t *be defined as in* (2) *with* $d = 0$ *and* δ *possibly different from zero. Then, the test based on the statistics* $R_b^i(d_0)$, ($R_b^i\left(\hat{d}\right)$), *rejects the hypothesis of* H_0 (H_0') *with probability approaching 1.*

5. Finite sample results.

This section presents the results of some Monte Carlo experiments designed to explore the finite sample performance of the test introduced in Section 4. Different DGP's have been considered (that will be detailed below) and, unless otherwise stated, innovations have been drawn from independent $N(0,1)$ distributions. The number of replications was set equal to 5000 in all cases.

The first experiment was to test the FI(d) (simple) hypothesis for several values of $d \in (0.5, 1.5)$, when the true model was generated as the sum of i.i.d innovations plus a time trend (with coefficient 0.05) and some deterministic terms that contained breaks at different time points ($\omega = \{0.20, 0.5, 0.80\}$), according to Models 0 to 3. Different sizes of breaks were considered, both in the constant and in the time trend. In particular, the size of the break in the constant was ξ_1 =$\{0.01,\ 0.05,\ 0.1\}$, and in the time trend was ξ_2 ={0.005, 0.01, 0.1}. No short-term semiparametric correction was introduced to compute the statistics in this case. Very remarkably, the power was equal to 100% in all cases, even for moderate sample sizes (T=100).

Next, the composite hypothesis $H_0' : d \in D_0$ with $D_0 = [1/2 + \varepsilon, 3/2 - \varepsilon]$ for a value of $\varepsilon = 0.01$, was considered for the same DGP as the one described above. As a first step, estimates of d were computed using the Feasible exact local Whittle estimator (FELW, henceforth) introduced by Shimotsu (2006) and Shimotsu and Phillips (2005) . The following table reproduces the mean and the standard deviation of the estimates of d when the true model is $i.i.d + trends$ and/or breaks.

[Table 5.1 about here]

This table illustrates the identification problem addressed in this paper: a short-memory sequence (even if it is *i.i.d*, as in this case) that contains trends and/or breaks that are not explicitly accounted for, tends to display positive estimated values of d.[8] Table 5.1 shows that the estimated values of d are far from the true value ($d = 0$) and are contained in the interval considered in this paper, (0.5-1.5). Even in the case where no break is present, that is, $\xi_1 = \xi_2 = 0$ (so that $y_t = 0.05t + \varepsilon_t$), the mean and standard deviations of d were 0.69 and 0.0353 for $T = 100$ and 0.743 and 0.027 for $T = 400$, respectively.

The second step is to compute the test with the values obtained in the previous estimation exercise. In agreement with the first experiment, when the estimated values of d are used to run the tests (in the case where no short-term autocorrelation) H_0' could always be rejected.

Next, short-term autocorrelation was introduced in the DGP. Tables 5.2. to 5.4 present the results of using Models 1-3 to test (simple) hypotheses of interest when the true DGP was an AR(1) process (with an autoregressive coefficient equal to 0.5), plus some breaks. Different locations of the break point were also tried ($\omega = \{0.20,\ 0.5, 0.80\}$) but, for the sake of brevity, only the figures corresponding to $\omega = 0.5$ are reported since they were all very similar.

[Table 5.2 about here]
[Table 5.3 about here]
[Table 5.4 about here]

From Tables 5.2 to 5.4 it is seen that the size of the break has not a big impact on power when d is chosen a priori. As expected, power improves when T and d_0 increase, since the test is consistent and the bigger d_0, the more distant H_0 and H_1 are.

Nevertheless, when d is estimated, the size of the break does matter. This is because the larger the break (or the coefficient in the trend component), the higher the estimated value of d. And, as Tables

[8]Similar results are obtained if other estimation methods are employed.

5.2 to 5.4 illustrate, the higher the value of d used to run the test, the higher the power obtained. The estimated values of d associated to the case where an AR(1) component was introduced are very similar to those reported in Table 5.1, that is, estimated values are around 0.7-1. To have an accurate picture of the power when an estimated d is used, one should compare the estimated values of d (Table 5.1) and the rejection rates reported in Tables 5.2 to 5.4. For instance, for the case where $\xi_1 = 0$ and $\xi_2 = 0.1$, Table 5.1 reports mean estimated values of $\hat{d}$ of 0.791 and 0.984 for $T = \{100, 400\}$ respectively. The rejection rates associated to these null hypothesis are approximately (see Table 5.3) 97.1% and 100%. This a very good approximation to the true rejection rates computed for each estimated d which delivered values of 96.9% and 100%, respectively.

Finally, we have compared the performance of the technique proposed here with other competing methods. To the best of our knowledge, the closest reference is Dolado, Gonzalo and Mayoral (2005), SB-FDF hereafter, who, by extending the popular Dickey-Fuller testing framework, are able to deal with a very similar problem. This technique is similar in spirit to Zivot and Andrew's (1992) but fractional orders of integration are allowed for. The tests is based on the t-statistic associated to y_{t-1} in a regression of $\Delta^d y_t$ on y_{t-1}, some deterministic components that are allowed to break and, possibly, some lags of $\Delta^d y_t$ to account for the serial correlation. When the date of the break is unknown, the test is computed over a sequence of possible break dates and then, the minimum of the t-tests is chosen. Several experiments have carried out to compare both techniques considering different scenarios: normal and non-normal innovations that may or may not be autocorrelated. When no short-term autocorrelation is included, both techniques deliver similar rejection rates, close to 100% for the relevant range of values of $d > 0.5$. These results are robust to the distribution of the innovations (normal, χ_1^2 and t_2). Nevertheless, when autocorrelation structure is allowed for, the technique proposed in this article is, in general, more powerful. Different DGP's have been considered, containing either AR or MA innovations or both. To facilitate the comparison, the following table reports the power of the SB-FDF test calculated with the same DGP as that used to compute Tables 5.2. to 5.4 above. For the sake of brevity, only the rejection frequencies computed with the values $\xi_1 = 0.05$ and $\xi_2 = 0.01$ are reported, since all were quite similar. The number of lags to be included in the SB-FDF regression was chosen according to the AIC.

[Table 5.5 about here]

6. Empirical Illustration

To illustrate the empirical applicability and the simplicity of the proposed technique, we now apply it to the study of inflation data. This variable occupies a privileged place in macro-econometrics since it plays a central role in the design of the monetary policy and has important implications for the behavior of private agents. Moreover, a new interest in the subject has arisen in the last few years that has motivated a large number of empirical and theoretical contributions. In spite of this great effort, there is no consensus in the literature about the most appropriate way to model the inflation rate. On the one hand, there is abundant empirical evidence that post-war inflation in industrial countries exhibits high persistence, close to the unit root behavior. The papers of Pivetta and Reis (2004) for the USA and O'Reilly and Whelan (2004) for the euro zone are some examples. On the other, some authors have argued that the above-mentioned results are very sensitive to the employed statistical techniques and that the observed persistence may be due to the existence of unaccounted breaks, probably stemming from changes in the inflation targets of monetary authorities, different exchange rate regimes or shocks in key prices. For instance, Levin and Piger (2003) have found evidence of a break in the intercept of the inflation equation and, conditional on this break, they argue that inflation shows very low persistence. Finally, Cogley and Sargent (2001, 2005) claim that non-stationary (integrated) representations of inflation are implausible from an economic point of view, since they would imply an infinite asymptotic variance, which could never be optimal if the Central Bank's loss function includes the variance of inflation. Then, they consider inflation as being a short memory ($I(0)$) process.

The aim of this section is to shed further light on this controversy by applying the techniques developed in this article. To facilitate the comparison with previous analysis, the same data set as in Pivetta and Reis (2004) has been employed: The price level, P_t, is measured through the seasonally-adjusted quarterly data on the GDP deflator from the first quarter of 1947 to the last quarter of 2003 (9 observations have been added with respect to their analysis). This data has been obtained from the Bureau of Economic Analysis. Then, inflation is computed as $\pi_t = 400 * \log(P_t/P_{t-1})$, that is, it is the quarterly continuously compounded annualized rate of change of the price level. Figure 1 presents a plot of this data.

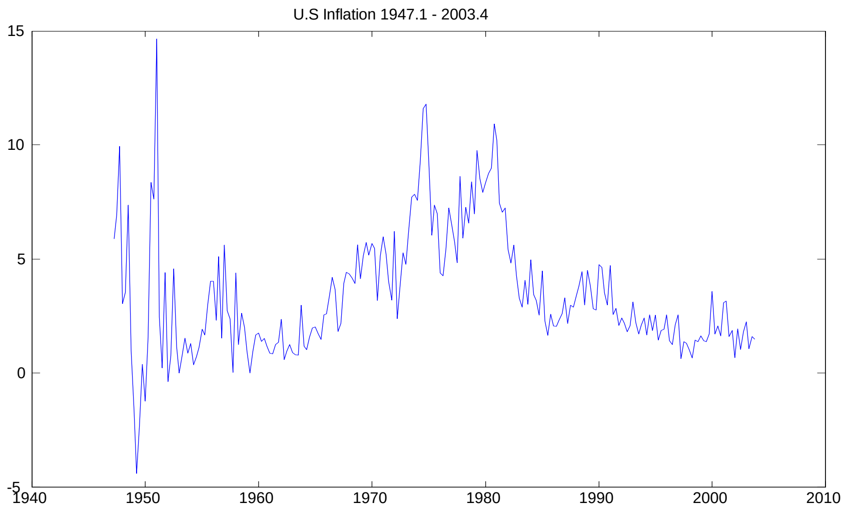

Figure 1

The contradicting results described above could be explained if the inflation rate was a FI process. Unit root tests are known to have very low power against FI alternatives. This could account for the non-rejection of the this hypothesis in some applications considering inflation. On the other hand, if inflation is FI and standard techniques for detecting and dating breaks are employed, it is well-known that spurious breaks are likely to be detected.

There is both economic and statistical support for the hypothesis of FI in inflation. Gadea and Mayoral (2005) provide an economic justification for the existence of fractional integration in inflation data. They consider a sticky price model as in Rotemberg (1987) and, by allowing for firms having heterogeneous costs of adjusting their prices, show that inflation behaves as a FI process. From an applied point of view, evidence in favor of FI behaviour in inflation has been reported in several papers (see, among others, Backus and Zin, (1993) , Baillie, Chung and Tieslau (1996) , Doornik and Ooms (2004), etc.). Nevertheless, the methods employed in those articles are not robust to the existence of structural breaks. Therefore, it remains to be checked whether the evidence supporting FI can be due to existence of structural breaks.

To begin the analysis, Table 6.1 presents the results of some standard tests for unit roots. The first two columns contain the figures from the Augmented Dickey-Fuller (ADF) and the Phillips- Perron (P-P) tests of $I(1)$ vs. $I(0)$ and the third one, those obtained by applying the KPPS test of $I(0)$ vs. $I(1)$. A constant was included as the only deterministic regressor to compute the tests.[9] From the first two columns it is seen that when the unit root model is tested against $I(0)$, the former is rejected. The opposite result is obtained when the hypothesis are reversed (third column). In this case, also the $I(0)$ is rejected against the alternative of $I(1)$.

[Table 6.1 about here]

The rejection of the $I(1)$ and the $I(0)$ hypotheses is compatible with the existence of both fractional integration and also with some types of structural breaks. This is so because unit root tests are known to have some power against the latter DGP's (see Lee and Schmidt (1996) , Diebold and Rudebush (1991) and Perron (1989)).

The next step is to test for the suitability of the FI specification. Table 6.2 presents the results of estimating d using different techniques: the Feasible Exact local Whittle estimators (FELW) (cf. Shimotsu, (2006) , Exact Maximum likelihood (EML, Sowell (1992)) and Minimun Distance (MD, Mayoral (2004)). In all cases, fractional values of d 'far' from both the I(0) and the $I(1)$ hypothesis are found.

[Table 6.2 about here]

[9] The number of lags for the ADF test was determined usin the AIC while the bandwidth for the P-P and KPSS test was chosen according to the Andrew's (1991) data dependent method.

Tests of fractional versus integer integration ($d = 0$ or 1) based on the values above are not able to reject the FI hypothesis at the 5% signification level, confirming previous findings about the existence of FI in inflation data (see the papers cited above).

Finally, it is checked whether the evidence in favor of FI is due to the existence of breaks in the intercept of the inflation equation as claimed by Levin and Piger, (2003). Since the true d_0 is unknown, we consider H_0' as null hypothesis, that is, the inflation rate is a non-stationary FI(d_0) process. In particular, $H_0' : d_0 \in D_0 = [1/2 + \varepsilon,\ 3/2\text{-}\varepsilon]$, for ε as small as desired. Table 6.3 presents the values of testing nonstationary FI against short-memory, allowing for a single break in the intercept. The estimates of d reported in Table 6.3. are used to compute the statistics.[10] The null hypothesis of non-stationary FI cannot be rejected against the alternative of $I(0)$+ breaks in any of the cases.

[Table 6.3 about here]

7. Conclusions

This paper analyzes the long-standing issue of determining the source of the non-stationarity observed in many economic variables: whether it is a result of a high degree of inertia (very persistent shocks) or it appears as a consequence of the existence of trends and/or of rare and unexpected events that are able to change the underlying structure of the series (breaks). We have extended the traditional approach of testing $I(1)$ versus $I(0)$+breaks by allowing for a richer class of persistent behaviors under H_0. In particular, the possibility of fractional integration has been explicitly taken into account. It has also been shown that explicitly considering FI processes is very relevant since tests of $I(1)$ vs. I(0) +breaks tend to reject the former hypothesis when the true DGP is a FI process with an integration order smaller than 1. The asymptotic properties of the tests statistics as well as their finite sample behavior have been analyzed. Finally, an empirical application that analyzes US inflation has been reported and evidence of FI behavior has been found in this data set. This finding helps to understand previous controversies.

[10] The bandwith to compute the Newey-West correction was chosen according to Andrews (1991). Different values were also tried and the results remain qualitatively identical.

References

Akonom J. and C. Gourieroux (1987) , “A functional central limit theorem for fractional processes”, Discussion Paper 8801, CEPREMAP, Paris.

Andrews, D. W. K. (1991) , “Heteroskedasticity and autocorrelation consistent covariance matrix estimation”, *Econometrica* 59, 817-854.

Andrews, D. W. K. (1993) , “Test for parameter instability and structural change with unknown break point”, *Econometrica* 61, N ^{o} 4, 821-856.

Andrews, D.W.K. and W. Ploberger, (1994), “Optimal tests when a nuisance parameter is present only under the alternative”, *Econometrica* 62, 1383-1414.

Bai, J. (1997), “Estimation of a change point in multiple regression models”, *The Review of Economics and Statistics*, Vol. 79, N^o 4, 551-563.

Bai, J. (1999) , “Likelihood ratio tests for multiple structural changes”, *Journal of Econometrics*, Vol. 91, N^o 2, 299-323.

Bai J. and P. Perron (1998) , “Testing for and estimation of multiple structural changes”, *Econometrica* 66, 47-79.

Baillie, R.T. (1996), “Long memory processes and fractional integration in economics and finance,” *Journal of Econometrics*, 73, 15-131.

Baillie R., Ch. F. Chung and M. Tieslau (1996), “Analyzing inflation by the fractionally integrated ARFIMA-GARCH model”, *Journal of Applied Econometrics*, 11, 23-40.

Backus, D. K, and and S. E. Zin, (1993), “Long memory inflation uncertainty: evidence from the term structure of interest rates”, *Journal of Money, Credit and Banking,* 25, 681-700.

Bhargava, A. (1986) , “On the theory of testing for unit roots in observed time series”, *The Review of Economic Studies* 53 (3) , 369-384.

Bhattacharya, R.N., Gupta, V.K., E. Waymire (1983), “The Hurst effect under trend”, *Journal of Applied Probability* 20, 649-662.

Beran, J. (1995) , “Maximum likelihood estimation of the differencing parameter for invertible and short and long memory autoregressive integrated moving average models”. *Journal of the Royal Statistical Society*, Series B, 57, No. 4, 659-672.

Berkes, I., L. Horvath and P.S. Kokoszka (2006),“On discriminating between long-range dependence and changes in mean”, *Annals of Statistics* Vol. 34, No. 3.

Cai Y. and M. Shintani (2005) , “On the long-run variance ratio test for a unit root”, Working Paper 05-W06, Vanderbilt University.

Cogley, T. and T. J. Sargent (2005), “The conquest of US inflation. Learning and robustness to model uncertainty”, *Review of Economic Dynamics,* Vol. 8, 2 , 528-563

Cogley, T. and T. J. Sargent (2001), "Evolving post World War II inflation dynamics". NBER Macroeconomics Annual 16, 331-373.

Davidson, J. (1994) , *Stochastic Limit Theory.* Oxford University Press.

Davidson, J. and P. Sibbertsen (2005), "Generating schemes for long memory processes: regimes, aggregation and linearity", *Journal of econometrics,* 128, 253-282.

Diebold, F.X. and A. Inoue (2001), "Long memory and regime switching", *Journal of Econometrics*, 105, 131-159.

Diebold, F.X, and G. Rudebush (1991) , "On the power of the Dickey-Fuller tests against fractional alternatives", *Economic Letters*, 35, 55-160.

Dolado J.J., J. Gonzalo and L. Mayoral (2002) , "A Fractional Dickey-Fuller test for unit roots", *Econometrica* 70, 5.

Dolado J.J., J. Gonzalo and L. Mayoral (2004) , "Testing for a unit root against fractional alternatives in the presence of a maintained trend", Mimeo.

Dolado J.J., J. Gonzalo and L. Mayoral (2005) , "What is what? a simple time-domain test of long-memory vs. structural breaks", Mimeo.

Doornik J. A and M. Ooms (2004), "Inference and Forecasting for ARFIMA Models With an Application to US and UK Inflation", Non linear dynamics in Econometrics, Vol. 8, 2 .

Elliott G., E. Rothemberg and J. Stock (1996) , "Efficient test for an autoregressive unit roots", *Econometrica* Vol. 64, N ^{o} 4.

Gadea, L. and L. Mayoral (2006), "The persistence of inflation in industrial countries", *International Journal of Central Banking*, Vol. 2, N ^{o} 1.

Geweke, J., and S. Porter-Hudak (1983), "The estimation and application of long memory time series models," *Journal of Time Series Analysis, 4*, 221-238.

Giraitis L., P. Kokoszka and R. Leipus (2001), "Testing for long memory in the presence of a general trend". *Journal of Applied Probability*, 38, No 4, 1033-1054.

Granger, C.W.J. (1980) , "Long memory relationships and the aggregation of dynamic models", *Journal of Econometrics* 14, 227-238.

Granger, C.W.J and K. Joyeux (1980) , "An introduction to long memory series", *Journal of Time series Analysis*, 1, 15-30.

Henry, M. and P. Zaffaroni (2002), "The long range dependence paradigm for Macroeconomics and Finance", in *Long range dependence: Theory and applications*, P. Doukhan, G. Oppenheim and M. Taqqu (ed). Birkhäuser, Boston.

Heyde, C. C. and W Dai (1996), "On the robustness to small trends of estimation based on the smoothed periodogram". *Journal of Time Series Analysis,* 17, 141-150.

Hosking, J. R.M. (1981) , "Fractional differencing", *Biometrika* 68, 165-176.

Hsu, C. C. (2001), "Change point estimation in regressions with I(d) variables", *Economics Letters* 70, 147-155.

Krämer W. and P. Sibbertsen (2002) , "Testing for structural changes in the presence of long memory", *International Journal of Business and Economics*, Vol.1, N ^{o} 3, 235-242.

Künsch, H. (1986) , "Discrimination between deterministic trends and long range dependence", *Journal of Applied Probability* 23, 1025-1030.

Lehmann E. L. (1959) , *Testing Statistical Hypothesis.* John Wiley and Sons: New York.

Levin A. and J. Piger (2003) , "Is inflation persistence intrinsic in industrial economics", Federal Reserve Bank of Saint Louis, Working Paper 2002-023.

Lee, D and P. Schmidt, (1996) , "On the power of the KPSS test of stationarity against fractionally-integrated alternatives", *Journal of Econometrics* 73, 285-302.

Marinucci, D. and P. Robinson, (1999) , "Alternative forms of fractional Brownian motion", *Journal of Statistical Planning and Inference*, 80, p. 111-122.

Mármol, F. and C. Velasco (2002) , "Trend stationarity versus long-range dependence in time series analysis", *Journal of Econometrics* 108, 25-42.

Mayoral, L. (2004) , "A new minimum distance estimator for ARFIMA processes", Working paper, Universitat Pompeu Fabra.

Nunes, L. C., C. M. Kuan and P. Newbold (1995) ,"Spurious Breaks", *Econometric Theory* 11, 736-749.

Perron, P., (2005), "Dealing with Structural Breaks", forthcoming in the *Palgrave Handbook of Econometrics*, Volume 1: Econometric Theory.

Perron, P. (1997) , "Further Evidence from Breaking Trend Functions in Macroeconomic Variables," *Journal of Econometrics* 80, 355-385.

Perron, P. (1989) , "The great crash, the oil price shock and the unit root hypothesis", *Econometrica* 58, 1361-1401.

Perron P. and Z. Qu (2004) , "An analytical evaluation of the log-periodogram estimate in the presence of level shifts and its implications for stock returns volatility". Mimeo.

Pivetta F. and R. Reis, (2004) , "The persistence of inflation in the United States", Mimeo.

Robinson, P.M. (1994), "Efficient tests on nonstationarity hypotheses". *Journal of the American Economic Association* 89, 1420-1437.

Rotemberg, J. (1987), "The New Keynesian Microfoundatios", *Macroeconomics Annual*, 2, 69-104.

Sargan J.D. and A. Bhargava (1983) , "Testing residuals from least squares regression for being generated by the gaussian random walk", *Econometrica* 51 (1) , 153-174.

Schmidt, P. and P.C.B. Phillips (1992), “LM Tests for a Unit Root in the Presence of Deterministic Trends”, *Oxford Bulletin of Economics and Statistics* 54, pp. 257-287.

Shimotsu, K (2006) , “Exact Local Whittle Estimation of Fractional Integration with Unknown Mean and Time Trend”, Queens Economics Department Working Paper No. 1061.

Shimotsu, K. (2005) . ‘Simple (but effective) tests of long memory versus structural breaks’, Mimeo.

Shimotsu, K. and P. C. B. Phillips (2000) , “Modified Local Whittle Estimation of the Memory Parameter in the Nonstationary Case”, Cowles Foundation Discussion Paper No.1265.

Sibbertsen P. and I. Venetis (2004), “Distinguishing between long-range dependence and deterministic trends”, Mimeo.

Sowell, F.B. (1992) : “Maximum likelihood estimation of stationary univariate fractionally-integrated time-series models,” *Journal of Econometrics, 53*, 165-188.

Tanaka, K. (1999), “The nonstationary fractional unit root,” *Econometric Theory*, 15, 549-582.

Teverovsky V. and M. Taqqu (1997) , “Testing for long-range dependence in the presence of shifting means or a slowly declining trend, using a variance-type estimator”, *Journal of Time Series Analysis*, Vol. 18, N ^{o} 3, 279-304.

Velasco, C. (1999*a*) , “Nonstatiory log-periodogram regression”, *Journal of Econometrics,* 91, 325-371.

Velasco, C. (1999*b*) ,“Gaussian semiparametric estimation of Nonstationary time series”, *Journal of Time Series Analysis*, Vol. 20, N ^{o} 1, 87-127.

Xiao, Z. and P.C.B. Phillips (1998) , “A primer on unit root testing“, J*ournal of Economic Surveys*, Vol. 12, N ^{o} 5, 423-469.

Zivot, E. and D.W.K. Andrews (1992), “Further evidence on the Great Crash, the oil-price shock and the unit root hypothesis”, *Journal of Business and Economic Statistics* Vol. 10, N ^{o} 3, 251-270.

Appendix A

Proof of Theorem 1

1. Consider first the case where the DGP is given by $y_t = \alpha + x_t$ and $\Delta^{d_0} x_t = u_t$ so that Z is a vector of ones. Then, the numerator of R^μ in (11) is given,

$$\sum_{t=1}^{T}(y_t - \hat{\alpha}_1)^2 = \sum_{t=1}^{T}\left(\alpha + x_t - T^{-1}\sum_{t=1}^{T}(\alpha + x_t)\right) \tag{21}$$

$$= \sum_{t=1}^{T} {x_t}^2 - T^{-1}\left(\sum_{t=1}^{T} x_t\right)^2, \tag{22}$$

where x_t is defined in (3). By the functional central theorem (see Akonom and Gourieroux, 1987) and the continuous mapping theorem, it follows that

$$T^{-2d_0}\sum_{t=1}^{T}(y_t - \hat{\alpha}_1)^2 = T^{-2d_0}\sum_{t=1}^{T} {x_t}^2 - \left(T^{-1/2-d}\sum_{t=1}^{T} x_t\right)^2$$
$$\xrightarrow{w} \lambda^2 (\int_0^1 (B^\mu_{d_0}(r))^2 dr$$

where $\lambda^2 = \sigma^2 \Psi(1)^2$ is the long-run variance, $B^\mu_{d_0}(r) = B_{d_0}(r)\, dr - \left(\int_0^1 B_{d_0}(r)\, dr\right)$ and B_{d_0} is a standard type II fractional Brownian motion (see Marinucci and Robinson, 1999).

For the case where $Z_t = \begin{pmatrix} 1 & t \end{pmatrix}$, it is known that (see Marmol and Velasco, 2002, pp. 38)

$$T^{-2d_0}\sum\left(y_t - \hat{\alpha}_1 - \hat{\beta}_1 t\right)^2 \xrightarrow{w} \lambda^2 (\int_0^1 (B^\tau_{d_0}(r))^2 dr$$
$$= \lambda^2\left(\int_0^1 B^2_{d_0}(r)\, dr - \left(\int_0^1 B_{d_0}(r)\, dr\right)^2 - 12\left(\int_0^1 (r - 1/2)\, B_{d_0}(r)\, dr\right)^2\right). \tag{23}$$

On the other hand, notice that under H_0, the process $\Delta^{d_0} y_t$ is an I(0) process plus a trend and therefore $\hat{\alpha}_0$, $\hat{\beta}_0$ are consistent estimator of α and β. This implies that the denominator of (11) tends in probability to the variance of u_t, γ_0. ■

Proof of Theorem 2

Notice that the numerator of (11) can be written as,

$$\frac{T^{-2\hat{d}}}{T^{-2d_0}} T^{-2d_0}\sum\left(y_t - \hat{\beta}_1' Z_t\right)^2 \tag{24}$$

and since $T^{-2\hat{d}}/T^{-2d_0} \to 1$ for any consistent estimator $\hat{d}$ of d_0, it follows that limit of (24) is identical to that of the numerator of (11) . With respect to the denominator, since $\hat{d} \xrightarrow{p} d_0$, a sufficient condition that guarantees that $T^{-1}\sum\left(\Delta^{\hat{d}}y_t - \hat{\beta}_0'\Delta^{\hat{d}}Z_t\right)^2 \xrightarrow{p} \gamma_0$ is uniform convergence of $T^{-1}\sum\left(\Delta^{\hat{d}}y_t - \hat{\beta}_0'\Delta^{\hat{d}}Z_t\right)^2$ on an open convex set B_0 containing d_0. This uniform convergence follows from the pointwise convergence of $T^{-1}\sum\left(\Delta^{d_0}y_t - \hat{\beta}_0'\Delta^{d_0}Z_t\right)^2$ to γ_0 and an equicontinuity argument using the compactness of D_0 and the differentiability of $\sum\left(\Delta^{d}y_t - \hat{\beta}_0'\Delta^{d}Z_t\right)^2$ with respect to d (cf. Davidson, 1994, p. 340, and Velasco and Robinson, 2000).■

Proof of Theorem 3

Under H_1, both the numerator and denominator represent the variances of (asymptotically) stationary processes and therefore they converge at rate T. This implies that the statistic tends to zero a rate T^{1-2d_0} if $d > 0.5$ and therefore, the probability of rejecting H_1 tends to 1 if $T \to \infty$.■

Proof of Theorem 4

The proof of this theorem is constructed along the lines of the proofs of Theorem 1 in Perron (1997) (P. henceforth) and also Theorem 1 in Zivot and Andrews (1992) (Z&A hereafter). We consider first the proof for the case where $u_t = \varepsilon_t$ is an i.i.d. sequence and then we will relax this assumption.

We adopt the same notation as in the above-mentioned papers. Let $S_t = \sum_{j=0}^{t-1}\pi_j(-d)\varepsilon_{t-j}$, $(S_0 = 0)$, and $X_T(r)$ be the partial sum process defined as,

$$X_T(r) = T^{1/2-d_0}\sigma^{-1}S_{[Tr]},\ \ (j-1)/T < r < (j+1)/T \text{ for } j = 1, ..., T,$$

where $\sigma^2 = p\lim_{T\to\infty} T^{-1}\sum_{t=1}^{T}\varepsilon_t^2$. Let us define $z_{tT}^i(\omega)$ for $i = \{0,1,2,3\}$ as the vector that contains the deterministic components for model i under the alternative hypothesis. For instance, if $i = 1$, $z_{tT}^1(\omega)' = \begin{pmatrix} 1 & t & DC_t(\omega) \end{pmatrix}$. Also, $Z_T^i(\omega, r)$ represents a rescaled version of the deterministic regressors, i.e., $Z_T^i(\omega, r) = \theta_T^i z_{[Tr]T}^i(\omega)$, where θ_T^i is a diagonal matrix of weights.[11] Finally, we define the limiting functions $Z^0(\omega, r) = (1, du(\omega, r))$ where $du(\omega, r) = 1_{(r>\omega)}$, $Z^1(\omega, r) = (1, r, du(\omega, r))$, $Z^2(\omega, r) = (1, r, dt^*(\omega, r))$ where $dt^*(\omega, r) = (r-\omega)1_{(r>\omega)}$ and $Z^3(\omega, r) = (1, r, du(\omega, r), dt^*(\omega, r))$. The test statistics can be rewritten as:

$$\inf_{\omega\in\Omega} R_b^i(\omega) = \inf_{\omega\in\Omega}\frac{\sum_{j=1}^{T}\left(y_t^i(\omega)\right)^2}{\sum_{j=2}^{T}\left(\Delta^{d_0}y_t^i\right)^2},\ \text{ for } i = \{0,1,2,3\}, \tag{25}$$

[11] For instance, in Model 1,

$$\theta_T^1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & T & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

where $y_t^i = y_t - z_{tT}^i(\omega)' \left(\sum_{s=1}^T z_{sT}^i(\omega) z_{sT}^i(\omega)'\right)^{-1} \sum_{s=1}^T z_{sT}^i(\omega) y_s$ for $i = \{0,1,2,3\}$, $\Delta^{d_0} y_t^0 = \Delta^{d_0} y_t - \hat{\alpha}\Delta^{d_0}$ and $\Delta^{d_0} y_t^i = \Delta^{d_0} y_t - \hat{\alpha}\Delta^{d_0} - \hat{\beta}\Delta^{d_0-1}$ for $i = \{1,2,3\}$. Henceforth, only Model 1 will be considered. Proofs for models {0,2,3} are analogous and therefore, are omitted. For simplicity, the superscript denoting the model is dropped henceforth.

The proof will be completed in three steps that closely follow Perron's and Z&A's approach. In the first one, it is shown that the numerator in (25) can be written as a functional g that is a composition of functionals depending on $X_T(.)$ and $Z_T(.,.)$. Next, some joint convergence results are needed and finally, it is shown that g is a composition of continuous functionals and then is continuous. The proof of the theorem is completed by applying the CMT.

First step. By expression $(A.3)$ in Perron (1997),

$$\begin{aligned} T^{-2d_0}\sum_{j=1}^{T}(y_t(\omega))^2 &= \sigma^2\int_0^1 \{X_T(r) - P_{Z_t}(\omega) X_T(r)\}^2 dr + o_{p\omega}(1) &(26)\\ &= \sigma^2 H_1(X_T,\ P_{Z_t}(\omega) X_T(r)) + o_{p\omega}(1), &(27)\end{aligned}$$

where $o_{p\omega}(1)$ denotes a random variable that converges in probability to zero uniformly in ω, and

$$P_{Z_t}(\omega) X_T(r) = Z_T(\omega,r)' \left[Z_T(\omega,r) Z_T(\omega,r)'\right]^{-1} Z_T(\omega,r)\int_0^1 Z_T(\omega,s) X_T(s)\, ds.$$

It follows that the numerator of (25) can be expressed as a functional g with arguments X_T and $P_{Z_t}(\omega) X_T(r)$, such that,

$$\inf_{\omega\in\Omega} T^{-2d_0}\sum_{j=1}^{T}(y_t(\omega))^2 = g(X_T, P_{z_t}(\omega) X_T(r)),$$

where,

$$g = h^*\left[H_1(X_T,\ P_{z_t}(\omega) X_T(r))\right],$$

with $h^*(m) = \inf_{\omega\in\Omega} m(\omega)$ for any real function $m = m(.)$ on Ω and H_1 is defined by (27).

Second step. By Akonom and Gourieroux (1987),

$$X_T(.) \xrightarrow{w} B_{d_0}(.) \tag{28}$$

and by a similar argument as that used in Lemmae A.2 and A.3 in Perron,

$$\begin{aligned} &P_{Z_T}(\omega) X_T(r) \xrightarrow{w} P_Z(\omega) B_{d_0}(.) &(29)\\ \equiv\ &Z(\omega,r)' \left[Z(\omega,r) Z_T(\omega,r)'\right]^{-1} Z(\omega,r)\int_0^1 Z(\omega,s) B_{d_0} ds. \end{aligned}$$

Furthermore, (28) and (29) hold jointly (see Perron, Lemma A.1).

Third step. The final step is to show continuity of various functionals. Continuity of h^* is shown in Z&A (Lemma A.4) while continuity of H_1 is shown in Perron (Lemma A.2, p. 383). This implies that g is also continuous. This result, combined with (28) and (29) and the CLT imply that the numerator of (25) converges to $\inf_{\omega\in\Omega}\left(\sigma^2\int_0^1\{B_{d_0}(r)-P_Z(\omega)B_{d_0}(r)\}^2 dr\right)$. On the other hand, it is straight forward to check that under H_0 and $u_t=\varepsilon_t$, then $T^{-1}\sum_{j=2}^{T}\left(\Delta^{d_0}y_t\right)^2\xrightarrow{p}\sigma^2$.

For the general case where u_t is allowed to present autocorrelation, using stardard results it is possible to obtain that the numerator of (25) tends to $\inf_{\omega\in\Omega}\left(\lambda^2\int_0^1\{B_{d_0}(r)-P_Z(\omega)B_{d_0}(r)\}^2 dr\right)$ whereas the denominator converges to γ_0. ■

Proof of Theorem 5

The numerator of $R_b\left(\hat{d}\right)$ can be written as,

$$\left(\frac{T^{-2\hat{d}}}{T^{-2d_0}}\right)T^{-2d_0}\inf_{\omega\in\Omega}(\sum_{t=1}^{T}(y_t-\hat{\beta}_1'Z_t-\hat{\delta}(\omega)'V_t(\omega))^2)$$

where the first factor converges to 1 for any consistent estimator $\hat{d}$ of d_0 and the distribution of the second term has been derived in Theorem 4. The denominator of $R_b\left(\hat{d}\right)$ is unchanged with respect to (11) and therefore the steps described in Theorem 3 apply.■

Proof of Theorem 6

The proof of this theorem can be constructed along the same lines as that of Theorem 3 and therefore is omitted.■

Appendix B

The asymptotic distributions defined in the main text have been simulated for 50 values of d, from $d = 0.51$ up to $d = 1.49$ with an increment of 0.02 between each consecutive values. The number of replications was 10000 and the innovations were independent Gaussian series.

The results are summarized in Tables B1 to B6 by means of the coefficients of polynomial OLS regressions of the 1%, 5%, and 10% sample quantiles of the corresponding statistic for each value of d on a polynominal of d, namely, $\begin{pmatrix} 1 & d & d^2 & d^3 & d^4 \end{pmatrix}$. These tables can be used to obtain critical values for any value of $d \in (1/2, 3/2)$ with great precision by replacing d by d_0 or $\hat{d}$.

TABLE B1

Critical values $\mathbf{R}^{\mu}$ test

Simplest case: $H_0 : \Delta^{d_0}(y_t - \alpha_0) = \varepsilon_t$; $H_1 : (y_t - \alpha_1) = \varepsilon_t$

T	$T = 100$					$T = 400$				
$S.L.$| regressors	c	d	d^2	d^3	d^4	c	d	d^2	d^3	d^4
1% S.L	10.26	-36.99	49.84	-29.60	6.53	15.18	-56.18	77.25	-46.26	10.40
5% S.L.	11.07	-39.94	53.19	-31.51	6.93	16.02	-58.87	80.54	-48.43	10.78
10% S.L	11.37	-40.46	54.05	-31.91	7.00	16.53	-60.48	82.53	-49.54	11.01

TABLE B2

Critical values R^{τ} test

Trended case: $H_0 : \Delta^{d_0}(y_t - \alpha_0 - \beta_0 t) = \varepsilon_t$; $H_1 : y_t - \alpha_1 - \beta_1 t = \varepsilon_t$

T	$T = 100$					$T = 400$				
$S.L.$| regressors	c	d	d^2	d^3	d^4	c	d	d^2	d^3	d^4
1% S.L	9.71	-35.08	47.27	-28.05	6.17	14.72	-54.67	75.31	-45.49	10.16
5% S.L.	10.53	-37.90	50.94	-30.19	6.64	15.53	-57.35	78.67	-47.38	10.55
10% S.L	10.97	-39.36	52.81	-31.26	6.87	16.08	-59.23	81.12	-48.80	10.86

TABLE B3

Critical values R_b^0 test

Model 0: $H_0 : \Delta^{d_0}(y_t - \alpha_0) = \varepsilon_t$; $H_1 : y_t - \alpha_1 - \alpha_2 DC_t(\omega) = \varepsilon_t$

T	$T = 100$					$T = 400$				
$S.L.$| regressors	c	d	d^2	d^3	d^4	c	d	d^2	d^3	d^4
1% S.L	9.042	-32.829	44.420	-26.452	5.839	14.14	-52.73	72.86	-44.12	9.87
5% S.L.	9.610	-34.745	46.886	-27.870	6.144	14.98	-55.64	76.69	-46.35	10.35
10% S.L	9.949	-35.886	48.353	-28.713	6.325	15.38	-57.00	78.44	-47.37	10.57

TABLE B4

Critical values R_b^1 test

Model 1: $H_0 : \Delta^{d_0}(y_t - \alpha_0 - \beta_0 t) = \varepsilon_t$; $H_1 : y_t - \alpha_1 - \alpha_2 DC_t(\omega) - \beta_1 t = \varepsilon_t$

T	$T = 100$					$T = 400$				
$S.L.$| regressors	c	d	d^2	d^3	d^4	c	d	d^2	d^3	d^4
1% S.L	8.667	-31.544	42.744	-25.478	5.628	13.91	-52.09	72.18	-43.80	9.81
5% S.L.	9.207	-33.390	45.137	-26.860	5.926	14.57	-54.30	75.00	-45.41	10.15
10% S.L	9.497	-34.366	46.390	-27.578	6.080	14.91	-55.43	76.41	-46.19	10.52

TABLE B5

Critical values R_b^2 test

Model 2: $H_0 : \Delta^{d_0}(y_t - \alpha_0 - \beta_0 t) = \varepsilon_t$; $H_1 : y_t - \alpha_1 - \beta_1 t - \beta_2 DT_t = \varepsilon_t$

T	$T = 100$					$T = 400$				
$S.L.$| regressors	c	d	d^2	d^3	d^4	c	d	d^2	d^3	d^4
1% S.L	9.084	-33.098	44.882	-26.766	5.914	14.34	-53.67	74.35	-45.11	10.11
5% S.L.	9.671	-35.097	47.461	-28.248	6.232	14.86	-55.22	76.13	-46.00	10.27
10% S.L	10.005	-36.221	48.896	-29.066	6.4076	15.23	-56.50	77.76	-46.93	10.47

TABLE B6

Critical values R_b^3 test

Model 3: $H_0 : \Delta^{d_0}(y_t - \alpha_0 - \beta_0 t) = \varepsilon_t$; $H_1 : y_t - \alpha_1 - \alpha_2 DC_t(\lambda) - \beta_1 t - \beta_2 DT_t = \varepsilon_t$

T	$T = 100$					$T = 400$				
$S.L.$| regressors	c	d	d^2	d^3	d^4	c	d	d^2	d^3	d^4
1% S.L	8.417	-30.743	41.771	-24.950	5.520	13.58	-50.92	70.64	-42.91	9.62
5% S.L.	8.887	-32.345	43.836	-26.135	5.774	14.21	-53.08	73.39	-44.46	9.95
10% S.L	9.135	-33.171	44.878	-26.722	5.898	14.50	-54.02	74.57	-45.11	10.08

TABLE 5.1

MEAN AND STD OF $\hat{d}$ (FELW)♠

True process $(H_1): y_t = \xi_1 DC_t(\omega) + \xi_2 DT_t(\omega) + 0.05t + \varepsilon_t; \varepsilon_t \sim i.i.d; \omega = 0.5$

T |		Model 1 ($\xi_2 = 0$)			Model 2 ($\xi_1 = 0$)			Model 3		
	$\xi_1 =$	0.01	0.05	0.1	0	0	0	0.01	0.05	0.1
	$\xi_2 =$	0	0	0	0.005	0.01	0.1	0.005	0.01	0.1
T=100		0.690 (0.035)	0.691 (0.035)	0.700 (0.035)	0.690 (0.035)	0.696 (0.036)	0.791 (0.071)	0.690 (0.035)	0.700 (0.036)	0.801 (0.072)
T=400		0.743 (0.026)	0.743 (0.027)	0.746 (0.028)	0.750 (0.030)	0.762 (0.034)	0.984 (0.031)	0.751 (0.030)	0.764 (0.035)	0.985 (0.031)

(♠) Standard deviations in brackets.

TABLE 5.2

MODEL 1: POWER R_b^1 TEST; S.L:5%.

True process $(H_1): y_t = \xi_1 DC_t(\omega) + 0.05t + 0.5y_{t-1} + \varepsilon_t; \omega = 0.5$

	T=100					T=400				
ξ_1/H_0 :	d_0=0.6	d_0=0.7	d_0=0.8	d_0=0.9	d_0=1.0	d_0=0.6	d_0=0.7	d_0=0.8	d_0=0.9	d_0=1.0
0.01	65.3%	89.5%	98.1%	99.1%	100%	86.3%	93.2%	98.3%	99.2%	100%
0.05	66.9%	89.3%	99.2%	99.1%	100%	86.3%	93.2%	98.6%	100%	100%
0.1	66.6%	90.7%	98.1%	99.0%	100%	86.7%	93.2%	98.2%	100%	100%

TABLE 5.3

MODEL 2: POWER R_b^2 TEST; S.L:5%.

True process $(H_1): y_t = \xi_2 DT_t(\omega) + 0.05t + 0.5y_{t-1} + \varepsilon_t; \omega = 0.5$

	T=100					T=400				
ξ_2/H_0 :	d_0=0.6	d_0=0.7	d_0=0.8	d_0=0.9	d_0=1.0	d_0=0.6	d_0=0.7	d_0=0.8	d_0=0.9	d_0=1.0
0.005	61.3%	85.5%	98.1%	99.1%	100%	85.3%	92.7%	97.3%	99.5%	100%
0.01	61.9%	86.3%	99.2%	99.1%	100%	85.2%	92.8%	97.6%	99.3%	100%
0.1	56.6%	84.7%	97.1%	99.0%	100%	81.2%	92.1%	97.3%	99.6%	100%

TABLE 5.4

MODEL 3: POWER R_b^3 TEST; S.L:5%.

True process $(H_1) : y_t = DC_t(\omega) + \xi_2 DT_t(\omega) + 0.05t + 0.5y_{t-1} + \varepsilon_t;\ \omega = 0.5$

	T=100					T=400				
$(\xi_1, \xi_2)\|H_0$:	d_0=0.6	d_0=0.7	d_0=0.8	d_0=0.9	d_0=1.0	d_0=0.6	d_0=0.7	d_0=0.8	d_0=0.9	d_0=1.0
(0.01,0.005)	63.3%	87.5%	97.1%	99.1%	100%	87.3%	92.5%	96.3%	99.6%	100%
(0.05,0.01)	63.9%	87.3%	97.2%	99.1%	100%	87.3%	92.5%	96.3%	99.8%	100%
(0.1,0.1)	56.6%	82.7%	97.1%	99.8%	100%	89.2%	93.0%	95.3%	99.7%	100%

TABLE 5.5

POWER SB-FDF TEST R_b^1 TEST; S.L:5%.

True process $(H_1) : y_t = \xi_1 DC_t(\omega) + 0.05t + 0.5y_{t-1} + \varepsilon_t;\ \omega = 0.5$

	T=100				T=400			
ξ_1/H_0	d_0=0.6	d_0=0.7	d_0=0.8	d_0=0.9	d_0=0.6	d_0=0.7	d_0=0.8	d_0=0.9
Model 1	79.3%	81.5%	87.1%	92.4%	84.3%	90.2%	93.3%	95.2%
Model 2	54.3%	50.3%	54.2%	57.6%	86.3%	93.2%	98.6%	100%
Model 3	56.3%	57.1%	59.8%	63.3%	86.7%	93.2%	98.2%	100%

TABLE 6.1

UNIT ROOT TESTS

	ADF	P-P	KPSS ($I(0)$ vs. $I(1)$)
Value of the test	-3.49**	-5.88**	0.67*
Critical Values (5%)	-2.87		0.463

*,**Rejection at the 5% and the 1% level, respectively.

TABLE 6.2

ESTIMATION OF d

	FELW	EML	MD
$\hat{d}$	0.56 (0.11)	0.61 (0.14)	0.58 (0.12)

(Standard errors in brackets)

TABLE 6.3

LR tests FI(d) vs I(0) with one break in the constant.

	$\hat{d}_{FELW} = 0.56$	$\hat{d}_{EML} = 0.613$	$\hat{d}_{MD} = 0.58$
LR test	1.592	0. 697	1.182
Crit. Values (S.L.:5%)	0.5671	0.3876	0.4889